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Discussion of
"RESONANT VIBRATION OF STEEL STACKS"

by E. A. Dockstader, W. F. Swiger, and Emory Ireland
(Proc. Paper 541)

E. A. DOCKSTADER,[1] W. F. SWIGER,[2] and E. IRELAND,[3] MEMBERS, ASCE—The general mechanism of vortex excited resonant vibration of individual tall stacks and similar structure seems well established. The assembly in technical literature of data which may be used reliably in the design of such structures, however, has just begun. Primary problems upon which additional information is urgently needed are:

Damping characteristics for various types of structures, including their foundations
Coefficients of lift for computation of exciting forces
Mutual interference or effects of several stacks
Effects of boundary layer tripping devices upon vortex formation

The data supplied by Mr. Cedric Marsh, concerning the damping observed on an aluminum tower in Switzerland, are most welcome, since they furnish damping characteristics of a structure of markedly different proportions and of different material than the stacks which have been reported previously. Internal damping in a welded aluminum structure should account for only a small part of the total values reported by Mr. Marsh. External damping for such a structure could originate in the foundation or from air drag. Air drag would be viscous in character and the damping decrement would be constant. The marked increase in the damping decrement with increasing amplitude strongly indicates the damping in this structure probably originated in the foundation. If this is true, variation in the damping decrement could be expected for different foundation conditions and more data covering a variety of foundations would be desirable before acceptance of a general value of 0.1 for the damping decrement.

Such data as are available on damping characteristics are limited presently to stacks supported upon their own foundations. Additional data covering stacks supported upon the roofs of boiler houses or other structures, would be of value in further analysis of this phenomenon.

Mr. Marsh reports what may be the first case in literature of the excitation of the second mode of vibration of a cantilever by vortex action. The fundamental frequency of the aluminum tower described occurred at a wind velocity of 6 to 10 miles per hour. The second mode of vibration of a cantilever of constant cross-section is 6.36 times the fundamental frequency. This would agree with the fact that he observes vibrations at a wind speed of approximately 40 to 45 miles per hour. If this is the second mode of vibration being excited, then there should be a modal point about 0.22 of the height down

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from the top of the stack, and the frequency should be in the order of 5 cycles per second.

The need for data on the coefficient of lift at these large Reynolds numbers is self-evident. Theoretical values of this coefficient range from 0.9 to 1.71. Experimentally determined values at Reynolds numbers below one million range from 0.61 to 1.13. Experimental data or reliable observations from which this coefficient can be computed at Reynolds numbers of interest in stack design are totally lacking. In this connection, the observations of Mr. Marsh that the amplitudes of vibration of the aluminum tower at winds of 40 to 45 miles per hour were only slightly greater than at winds 6 to 10 miles per hour, and that the energy input at this higher velocity is of the same order, are most interesting. It is hoped that the data and reasoning leading to this conclusion will be made available in some other publication.

Mr. W. Watters Pagon, in his excellent discussion which covered both this paper and its companion paper, Proceedings Separate No. 540, brings to the fore the question of multiple stack installations. Data advanced by Mr. Pagon, together with his observations in Baltimore (4), indicate that in a quartering wind, or for wind along the line of a group of stacks, the mechanism of excitation may be markedly different than for a single stack. Additional data showing marked excitation of the second of two stacks in line, together with marked directional effects, have recently become available (10).

Exciting winds at Moss Landing were in a direction almost normal to the line of stacks. The observations indicated there was no coupling of the stacks under these conditions. Stacks 2 and 3 vibrated at the same frequency, but motion was completely independent. At times, only one of the two stacks would be moving; at other times, both. When they were both in motion, movement was sometimes in phase, and sometimes out of phase. It is, perhaps, significant that stack No. 1, which was completed except for the top eight feet high ring, did not move. This would indicate that, for a line of stacks spaced about 4.5 diameters and a wind normal to the line, there is no mutual interference.

Connecting the stacks at Moss Landing with a horizontal beam as suggested by Mr. Pagon was considered. Since experience indicated they could vibrate in phase as readily as out of phase and considering the definite self-excitation characteristics described by Steinman (2), it was believed that a simple horizontal beam system would not be effective in preventing vibration.

Additional observations on multiple stack installations would be most welcome, since installations with two, three, four or more stacks in line are quite common. If the mechanism of excitation for quartering winds or for wind along the line of stacks is different from that of a single stack, or for winds nearly normal to the line of stacks, the methods of analysis and data presently developed for single stacks could give misleading results when applied to multiple stack installations.

The possibility of devices for tripping the boundary layer, which might prove effective in preventing vortex induced motion, is again mentioned because of the probable economy such a device would offer as compared with designs based on non-resonance to extremely high wind velocities or on damper systems. There are theoretical reasons for believing that such boundary layer trippers might be effective, but, until supported by experimental data, dependence certainly could not be placed upon them for structures as important and expensive as the stacks for major power or industrial plants.

REFERENCE

10. Ozker and Smith "Factors Influencing the Dynamic Behavior of Tall Stacks Under the Action of Wind". To be published in Transactions ASME.

Discussion of
ANALYSIS OF CONTINUOUS STRUCTURES BY JOINT ROTATION

by Charles T. G. Looney
(Proc. Paper 679)

R. W. STEWART,[1] M. ASCE—If any point in a continuous beam or continuous frame is rotated by an externally applied moment there will be a series of moments and rotations generated throughout the entire structure. Various methods have been devised for computing this series of moments. The author has demonstrated one way of doing it but only for simple cases and by a rather complex and tedious computation.

Fig. n+1 does it a fast and easy way. In these traverse diagrams are found not only the desired series of moments but also their appurtenant series of joint rotations.

Fig. n+1 (a) gives the same series of moments found in the bottom line of the author's Fig. 3. Fig. 1+n (b) gives the series for an unbalanced unit

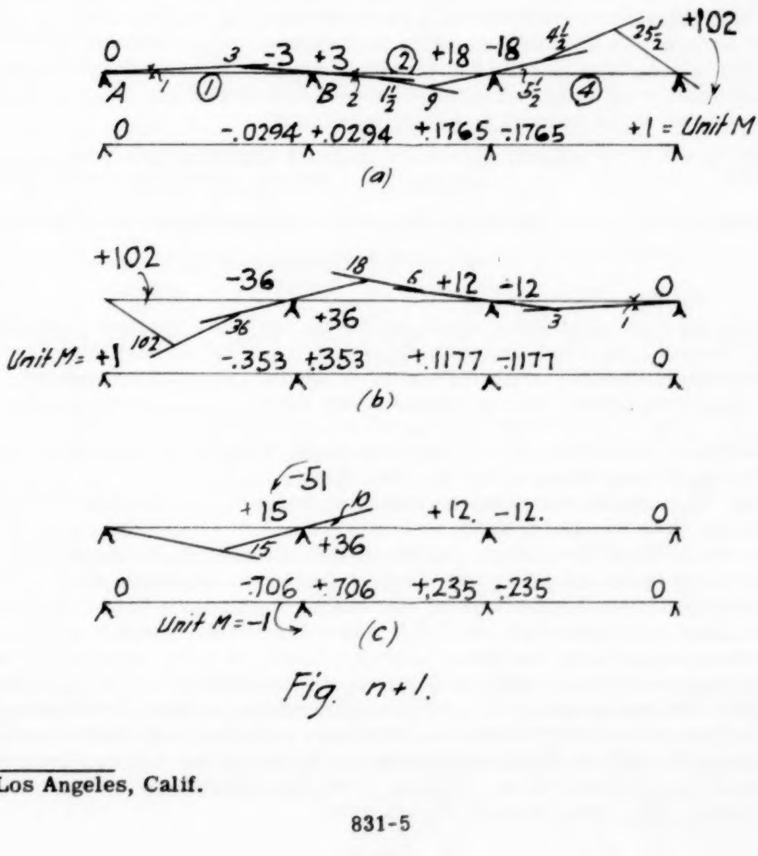

Fig. n+1.

1. Los Angeles, Calif.

moment at A found in line 3 of the author's Fig. 3. Fig. n+1 (c) gives the series for a moment applied at joint B.

The effect of unbalanced moments of -100 and +100 at A and B respectively is obtained by multiplying the traverse diagrams of Figures n+1 (a) and n+1 (c) by 102/100 and 51/100 respectively, or, as the author prefers, by multiplying the effect of unit moments at A and B by 100. This will give a duplication of the tabulation in the author's Fig. 4.

Since the author's computations do not include the geometry of the flexure they do not get the automatic check by Maxwell's Theorem of Reciprocal Deflections which is a feature of traverse computations.

The author in his last paragraph mentions some limitations which restrict the application of his method. With the Traverse Method, which carries the geometry of the flexure hand in hand with its mechanics, there are no such limitations. It can easily be applied to frames which are much more difficult to analyze than Fig. 5 in the Paper.[2,3]

J. J. POLIVKA,[4] M. ASCE—The method presented by Prof. T. G. Looney has several advantages as compared with the moment-distribution method. According to the latter the unbalanced fixed-end moments at each support are gradually distributed in a longer series of operations with infinitely increasing accuracy. The joint rotation method determines final exact distributed values of unbalanced moments, thus obtaining characteristic values of a given continuous structure which can be used for any scheme of loading. This simplification not only saves time in the analysis but also leads to absolute accuracy of the solution, which is of greater importance for reliable checking than for design, since any method of analysis, being based on certain assumptions, is only approximate. As demonstrated in Figs. 1, 2 and 3, the unity moment at the Support A is distributed to B C D :

	51:51 = 1	18:51 = 6/17	6:51 = 2/17	0
or	1.0000	0.3529	0.1177	0

Similarly the unity moment at D is distributed to other supports as follows:

	0	3:102 = 1/34	18:102 = 3/17	1
or	0	0.0294	0.1765	1.000

It can be shown that these ratios determine two characteristic (or fixed) points in each span which immediately lead to a simple moment diagram for any scheme of loading, no matter which, or how many, spans are loaded. This simplified method was presented by the writer in many publications.[5]

2. Die Traversen-Methode, Stewart-Kleinlogel, Wilhelm Ernst & Sohn Verlag, Hohenzollerndamm 169, West Berlin.
3. The Traverse Method in stress Analysis, Ralph W. Stewart, 1200 Arapahoe St., Los Angeles 6, Calif.
4. Doctor of Eng., Cons. Eng., Berkeley, Calif., Lecturer, Stanford Univ., formerly Research Associate, Univ. of California, Berkeley, Calif.
5. Graphical Methods of Analyzing Statically Indeterminate Structures, mimiogr. Lectures, Univ. of Calif., 1940, 2nd ed. '42; Frames and Bents in Reinforced Concrete, Revue du Beton Arme, Brussels, vol. 9, 10, 1920; Continuous Arches on Elastic Supports, Der Brueckenbau, vol. 4, 5 and 6, 1920; Determination of Fixed Points (Expressing Angular Rotations of Continuous Structures), Bulletin Technique de la Suisse Romande, p. 245, Lausanne, 1916; Analysis of Hyperstatic Frame Structures by Theorem of Fixed Lines, Techn. Univ., Prague, 1934; Designing Rigid Frames of Timber, Eng. News Record, Dec. 3, 1942.

One of the disadvantages of the Hardy Cross method of moment distribution is that the checking of accuracy of the solution can be performed only after the gradual (theoretically infinite) distribution is completed, and, if found to be unsatisfactory, the whole procedure must be repeated from the very beginning.

In the author's paper the joint rotations are determined by successive adjustments from support to support according to elastic yielding in the adjacent spans. Full restraint at the interior supports is assumed and free end supports are taken into consideration. Similar procedure would be followed if the end supports were fully or partially restrained.

It might be of interest to compare the author's method with some of the direct methods as presented and discussed by the writer in publications under [5,6] and [7]. In these methods the reverse value of relative stiffness $\underline{K} = \underline{I/L}$, or $\underline{K} = \underline{EI/L}$ is used, termed "elastic weight" or "elastic gravity": $\underline{G} = \underline{L/I}$, or $\underline{G} = \underline{L/EI}$, and for easier checking it is convenient to sketch the structure to scale (Fig. 6). Marking the spans $\underline{L}_1$, $\underline{L}_2$ and $\underline{L}_3$ and the distances of the characteristic (or fixed points from the left and right support $\underline{u}$ and $\underline{v}$, the distances $\underline{u}_1$ and $\underline{v}_3$ are equal to zero, and those of definite value are:

	v_1	u_2	v_2	u_3
	$6/23\ L_1$	$1/7\ L_2$	$1/4\ L_2$	$3/20\ L_3$
resulting from the ratios	$18:(18+51)$	$3:(3+18)$	$6:(6+8)$	$18:(18+102)$

By the writer's method (without successive moment distribution) identical values are obtained:

$$\frac{1/2\times 1/3}{1/2+5/36}L_1 \quad \frac{1/4\times 1/3}{1/3+1/4}L_2 \quad \frac{1/4\times 1/3}{1/4+1/12}L_2 \quad \frac{1/2\times 1/4\times 1/3}{1/8+11/72}L_3$$

Direct determination of the characteristic points $\underline{u}$ and $\underline{v}$ is very simple. Assuming a unit moment applied at the support $\underline{A}$, then $\underline{u}_2$ is the center of rotation of both adjacent spans, $\theta_1' + \theta_2^u = G_1/3 + G_2/2 = 1/3 + 1/4 = 7/12$, and $u_2 = \theta_2^u \cdot L_2/3 : (\theta_1' + \theta_2^u) = 1/4 \ .\ L_2/3 : 7/12 = 1/7\ L_2$. Similarly the characteristic point u_3 is the center of both members joining at the support $\underline{C}$: $\theta_2'' + \theta_3^u = G_2/6(3 - 1 : 6/7) + G_3/2 = 11/72 + 1/8 = 5/18$, and $u_3 = \theta_3^u \cdot L_3/3 : (\theta_2'' + \theta_3^u) = 1/8 \ .\ L_3/3 : 5/18 = 3/20\ L_3$. Simple graphical determination of these characteristic points is shown in Fig. 6. The bending moments

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6. Graphical Analysis of Continuous Beams with Variable Moments of Inertia, Zeitschrift fuer Betonbau, No. 4 and 5, 1917; Analysis of Continuous Beams on Elastic Supports, Techn. Obzor, 1917; Analysis of Triangular Bents, Beton & Eisen, No. 4 and 5, 1917; Graphical Analysis of Rectangular Bents, Armierter Beton, No. 9 and 19, 1917; Graphical Analysis of Closed Rectangular Frames, Techn. Obzor, No. 27 and 28, 1919.
7. Discussions of papers in Transa. ASCE: Wedge-Beam Framing, vol. 117 ('52), No. 2508; Continuous Arches and Bents Analyzed by Column Analogy, vol. 115 ('50), No. 2408; Analysis of Statically Indeterminate Structures Using Reduced Equations, vol. 111 ('46); Analysis of Rigid Frames by Superposition, vol. 110 ('45), No. 2262; Amplified Slope Deflection, vol. 110 ('45), No. 2260; Analysis of the General Two-Dimensional Framework, vol. 110 ('45), No. 2249; An Investigation of Steel Rigid Frames, vol. 107 ('42), No. 2130; Analysis of Building Frames with Semi-Rigid Connections, vol. 107 ('42), No. 2152.

for any type of loading either in an individual span or simultaneously in all spans can be determined very simply by graphical means (Fig. 7), and if absolute accuracy is required, can be checked algebraically.

The same method and procedure can be used for any type of framework, as is demonstrated by the example shown in Fig. 5. Elastic rotations are expressed again by characteristic points, u and v. In the beam member AB, u_1 is the center of rotation of both columns at A and of the beam member AB, assumed to be restrained at A: θ' of the columns is $1/4 : 2 = 1/8$ and θ_1^u of the member AB is $G \times 1/2 = 1/4 \,.\, 1/2 = 1/8$, and $u_1 = 1/8 \,.\, L_1/3 : (1/8 + 1/8) = 1/6\ L_1$. Similarly the characteristic points of other spans are determined as follows (Fig. 8):

Span L_2: Elastic rotation of AB at B: $\theta''_1 : G_1/6\ (3 - 1{:}5/6) = (1/4 \,.\, 9/5) : 6 = 3/40$. Elastic rotation of one column at B is equal to 1/8, and of both columns, since they are symmetrical and equal, 1/16. The total rotation at the joint B is therefore $\theta'_2 = 1 : (16 + 40/3) = 3/88$. Elastic rotation of the beam member BC is $1/4 \,.\, 1/2 = 1/8$, and $u_2 = 1/8 \,.\, L_3/3 : (1/8 + 3/88) = 11/42\ L_2$, which checks with the author's computation : $44L_2 : (124 + 44) = 11/42L_2$; $L_2 - u_2 = 31/42L_2$.

Span L_3: Elastic rotation of BC at C : $\theta''_2 = 1/24\ (3 - 1{:}31/42) = 17/248$. Combined with rotations of the columns, the total rotation at C is $\theta'_3 = 1 : (16 + 248/17) = 17/520$, and $u_3 = 1/8 \,.\, L_3/3 : (1/8 + 17/248 = 65/246\ L_3$, which checks with the author's value 260 : (260 + 720) = 65/246. Because of symmetry $v_1 = u_4$, and $v_2 = u_3$.

Span L_4: $\theta_3 = 99/1448$, the total rotation at D is $\theta_4 = 1 : (16 + 1448/99)$, and $u_4 = 1/24 : (1/8 + 99/3032) = 479/1434$, which conforms to the author's value 1516 : (422 - 1516) = 379/1434.

In the author's example of two-story framework, bending moments are calculated under uniform loading of each individual span, the max. moments on single, unrestrained spans being $M_o = +1/5$. The moments distributed to each member at the joint are proportional to reversed values of elastic rotations (actual rigidities). Considering the span L_1 under uniform load w, $M_o = 1/8\ w\ L_1^2 = 1/5$. The resulting bending moments at the joints A and B, M_A and M_B, will be distributed to all joined members in proportion to the actual rigidities. When a moment $M_A = 1$ is applied at the joint A, the elastic rotation of member AB becomes $\theta'_1 = G_1/6\ (3 - 1 : (1 - 379/1434\ ) = 577/8440$, and the elastic rotation of each column is $\theta'_1 = \theta''_I = G_I/4$. The actual stiffnesses are reversed valued of elastic rotations, and their sum is $\Sigma K = 8440/577 + 4 + 4 = 8 \,.\, 1592/577$. The actual carry-over factors are: Beam at A : (8 . 1055/577 : 8 . 1592/577) = 1055/1592; each column at A: 4 : (8 . 1592/577) = 577: 2 . 1592. Their proportion becomes 2110 : 577, which is in conformity with the author's ratio 4220: 1154.

Direct determination (calculation or graphical solution) is demonstrated in Fig. 9. Calculated values of the moments distributed at the joint A are:

$M_A = -169/408 = -0.41412$ (author's value - 0.414), $M_B = -379/408 = -0.92892$ (-0.929). M_B' is then distributed to the beam member $\underline{BC}$ and to both columns. Using the same procedure as for joint $\underline{A}$, the following values are obtained for the joint $\underline{B}$: Left end of the beam member $\underline{AB}$, $M_B'' = -181/408 = -0.44363(-0.444)$, right end of $\underline{AB}$, $M_C' = +65/408 = +0.15931(0.160)$.

The moment at $\underline{B}$, $M_B = -379/408$, is carried over to the beam member $\underline{BC}$ with the ratio 181/379, $M_B' = -181/408$, and to the columns with the ratio 99/379, $M_{II}'' = -99/408$. The elastic rotations of the beam and both columns are equal, 99/3264. The same rotation must have the beam member $\underline{AB}$ at the joint $\underline{B}$: $\theta_B'' = 1/8(1 - 309/408) = 99/3264$.

Fig. 10 shows the extreme simplicity of graphical solution developed by the writer.[7] The point $D_{1,I}$ is the center of rotation of all members joined rigidly at $\underline{A}$, produced by a moment at $\underline{A}$ under the assumption that the opposite ends of the members are freely supported. For unit moment at $\underline{A}$, the rotation of the beam $\underline{AB}$ is $G_1/2 = 1/4 \,.\, 1/2 = 1/8$, and is concentrated at the third-point I_1'. Similarly, the rotation of the lower column, $\underline{AA'}$, is $G_I/2 = 1/2$, and is concentrated at the upper third-point of the column, I_I''. In this special case the upper restraining column, AA'', is identical with the lower column, AA', and has the same restraining effect in any position (when the sidesway is not considered), it can be assumed to be turned 180 degrees and to coincide with the column AA'. The resulting rotation of both columns is then equal to 1/4. Applying these elastic rotations as weights acting at the third-points $I_1'(1/8)$ and at $I_I''(1/4)$, the resulting gravity center $D_{1,I}$ is found.

In a similar way, elastic gravity centers of other frame bays are determined ($D_{1,II}$, $D_{2,II}$, $D_{2,III}$, etc.). Fully restrained columns have centers of elastic rotations at the lower third-points (I_I', I_{II}', etc.). It can readily be proved that the line $I_I'D_{1,II}$ intersects the beam axis $\underline{AB}$ at the left characteristic point I_1'. The left characteristic point of the second span, I_2', is found as follows: The line $D_{1,II}I_1'$ intersects the perpendicular line through the right third-point of the first span at the point $I_{1,0}''$. Since both beam members $\underline{AB}$ and $\underline{BC}$ have the same relative stiffness, the line $I_{1,0}'' I_{2,0}$ has to pass through $\underline{B}$, and since $L_1 = L_2$, the distances of the points $I_{1,0}''$ and $I_{2,0}'$ from the beam axis are equal. The line $I_{2,0}'D_{2,II}$ intersects the axis $\underline{BC}$ at the left characteristic point I_2'. The left characteristic points of the third and fourth span are determined similarly. Because of symmetry of the frame work right characteristic points are simultaneously found and it is not necessary to proceed in this way from the right end $\underline{E}$.

This simplified method can be used for any degree of restraint of the columns. If the supports are hinged, elastic rotations of the columns are equal to G/3 (instead G/4 considered for full restraint).

Fig. 9 shows a graphical determination of bending moments of the two-story frame wherein only the first span carries uniform load. For checking purposes the same bending moment for simply supported span $\underline{AB}$ as in the author's example ($M_o = +1.5$) is assumed. Vertical lines through characteristic points, I_1' and I_2', intersect the lines $\underline{AT}$ and $\underline{BT}$ at points which determine the moment closing line (the line of negative moments). These moments are distributed to the adjacent members in proportion to the reversed values of elastic rotations at each particular joint.

The accuracy of the solution can be checked at each joint by calculating actual rotations of the members, which must be equal. Thus, at the joint $\underline{B}$ the elastic rotations are found as follows:

Member $\underline{AB}$: $1/8 - \dfrac{103}{136 \times 8} = 33/1088$

Member $\underline{BC}$: $\dfrac{164 - 65}{408 \times 8} = 33/1088$

Member $\underline{BB'}$: $\dfrac{99}{408 \times 8} = 33/1088$

Similarly, elastic rotations of individual members joined at other connections must be equal and have the following values: At $\underline{A}$ --- 169/3264; at $\underline{C}$ --- 17/3264; at $\underline{D}$ ---3/3264 and at $\underline{E}$ ---1/3264.

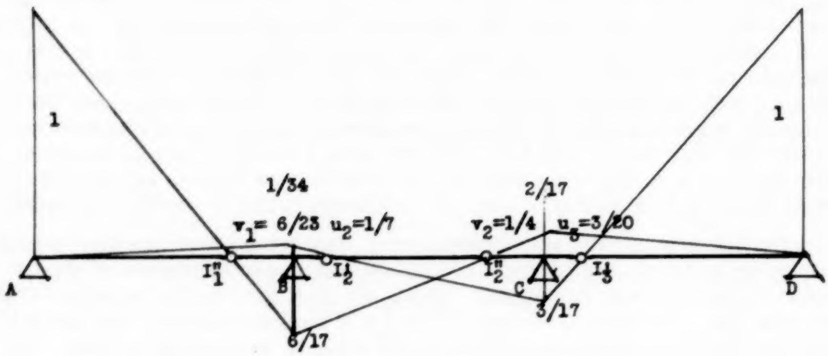

Figure 6.

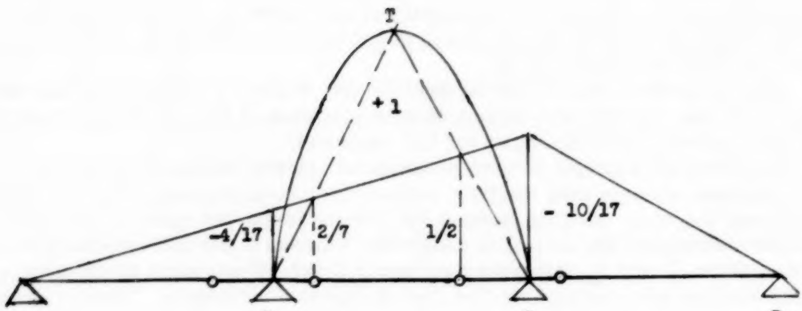

Figure 7.

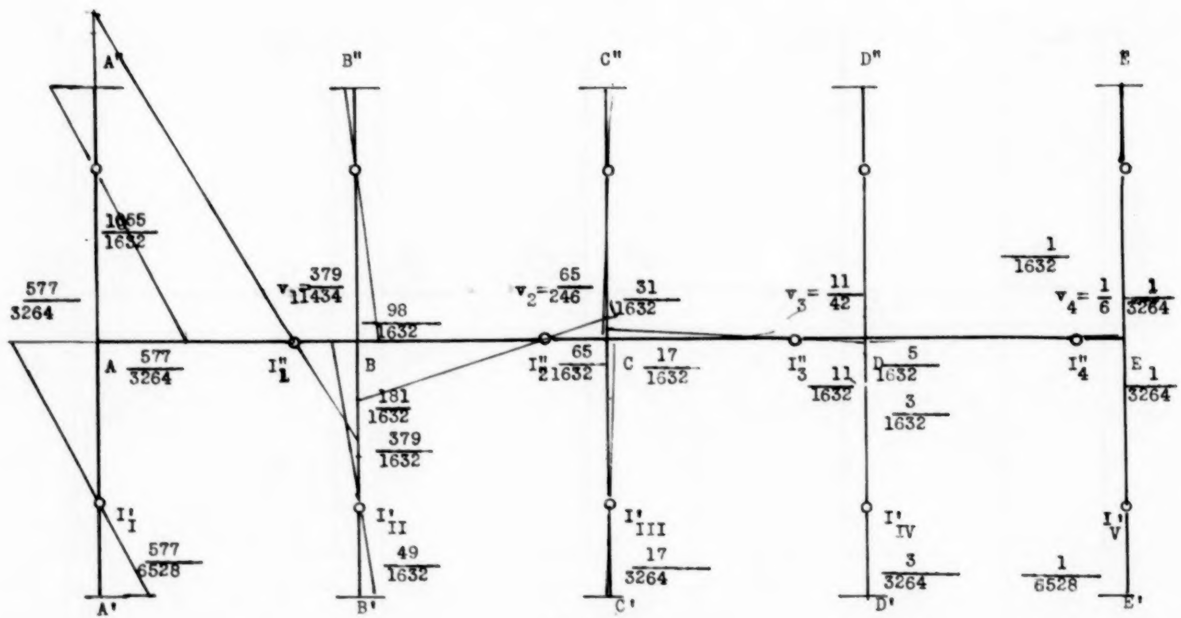
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Figure 8.

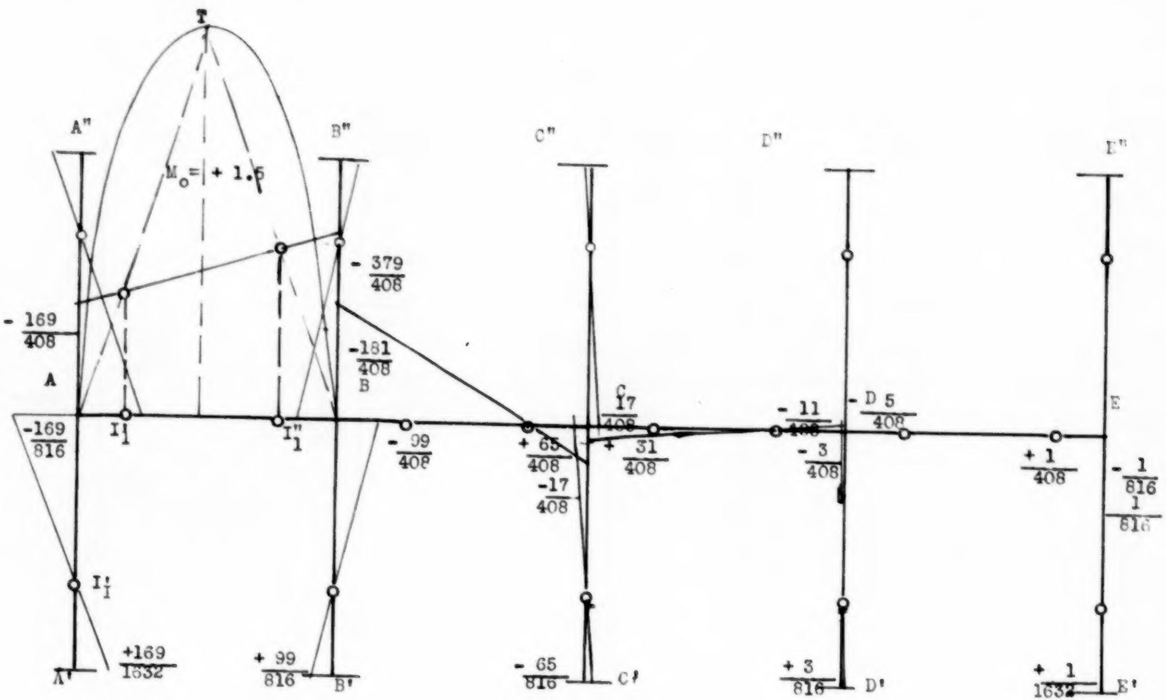
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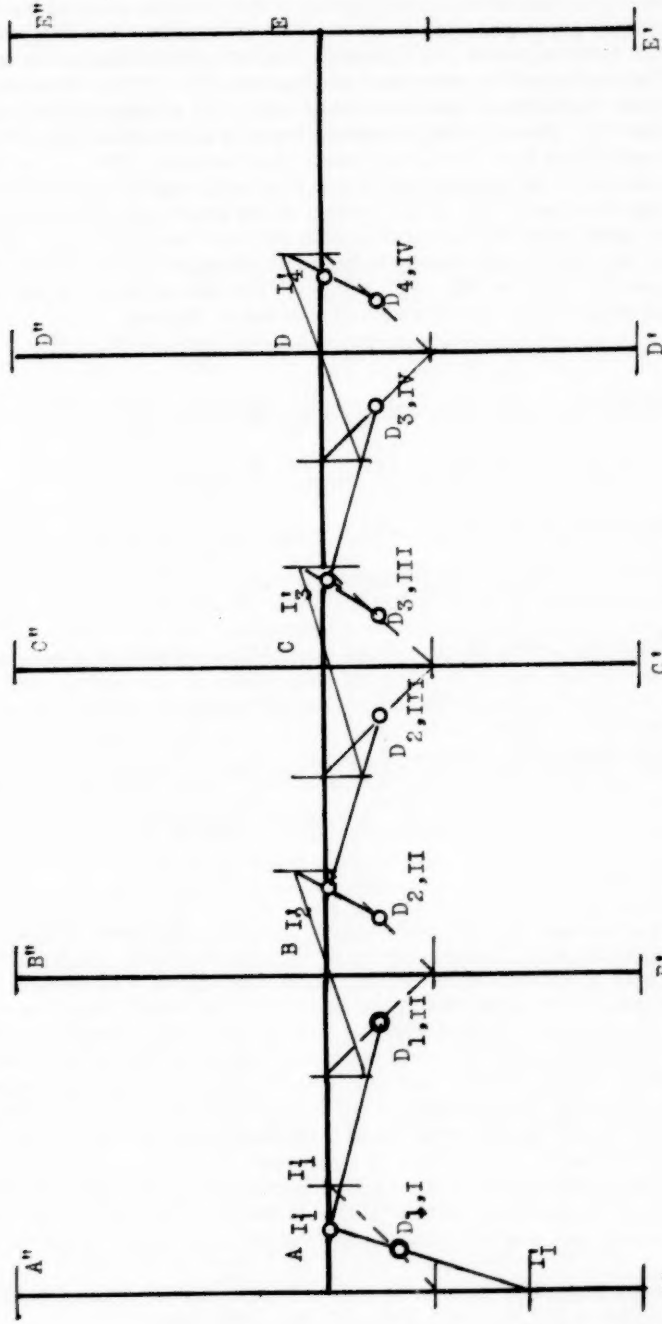
Figure 9.



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$D_{1,I}$
$D_{1,II}$
$D_{2,II}$
$D_{2,III}$
$D_{3,III}$
$D_{3,IV}$
$D_{4,IV}$

Figure 10.

HARRIS SOLMAN,[8] A.M. ASCE—The solution of continuous beams requires a determination of the end moments of the various spans at the joints. For convenience each of the end moments may be expressed as M_{jk}, which refers to the moment at end j of span j-k, j and k representing joints and being usually expressed by numerical designations ($k = j \pm 1$). For any joint j, M_{jk} is equal to the fixed-end-moment of span j - k at end j (which may be designated as M'_{jk}) plus a linear algebraic form in terms of all the joint rotations each multiplied by a coefficient which is a function of the characteristics of the beams of the system but is invariant with regard to the loading. It is that algebraic form that is the subject of the paper under discussion.

Since the joint rotations are produced by the fixed-end-moments about the joints, they may well be expressed in terms of those moments. Thus the rotation of joint j is equal to $M'_{j(j-1)} - M'_{j(j+1)}$. The end moments at the various joints in the system may therefore be expressed as follows:

$$M_{10} = M'_{10} + b_{11}(M'_{10} - M'_{12}) + b_{12}(M'_{21} - M'_{23}) + \cdots$$
$$\cdots + b_{1n}\left\{M'_{n(n-1)} - M'_{n(n+1)}\right\}$$

$$M_{12} = M'_{12} + a_{11}(M'_{10} - M'_{12}) + a_{12}(M'_{21} - M'_{23}) + \cdots$$
$$\cdots + a_{1n}\left\{M'_{n(n-1)} - M'_{n(n+1)}\right\}$$

$$M_{21} = M'_{21} + b_{21}(M'_{10} - M'_{12}) + b_{22}(M'_{21} - M'_{23}) + \cdots$$
$$\cdots + b_{2n}\left\{M'_{n(n-1)} - M'_{n(n+1)}\right\}$$

$$M_{23} = M'_{23} + a_{21}(M'_{10} - M'_{12}) + a_{22}(M'_{21} - M'_{23}) + \cdots$$
$$\cdots + a_{2n}\left\{M'_{n(n-1)} - M'_{n(n+1)}\right\}$$

These expressions for each end moment in terms of a fixed-end-moment at the same point plus a linear function of all joint rotations are not new. They have been presented by the writer some time ago in a discussion of Matrix Analysis,[9] and were expressed there in an identical form (See equation 131). The author's contribution consists in devising a method for deriving the coefficients for any given system of beams by means of a novel and interesting single analysis and also in attributing physical significance to these coefficients. Specifically, from the author's analysis b_{jj} and a_{jj} evolve as the "RMS" (Relative Modified Stiffness) to the left and to the right of joint j respectively each divided by their sum. For $i < j$, a_{ij} and b_{ij} are each equal to b_{jj} multiplied by the ratio of the "C O F to the left" at point i(i+1) and i(i-1) respectively to the "C O F to the left" at point j(j-1). For $i > j$, a_{ij} and b_{ij} are each equal to a_{jj} multiplied by the ratio of the "C O F

8. Div. Bridge Eng., U.S. Bureau of Public Roads, Div. 1, Albany, N. Y.
9. Transactions A.S.C.E., 1947, Vol. 112; pp. 1142-1150.

to the right" at points i(i+1) and i(i-1) respectively to the "C O F to the right" at point j(j+1). In the writer's presentations of the formulas for end moments[9] the coefficients were set up in a general form for any system and were shown to be built up for a system of any order, without the need of an analysis, from the coefficients of the next lower order. By applying the writer's coefficients, as set up in equation 133, to the example used by the author, which involves a system of the second order (n = 2), the equations become

$$M_{10} = M'_{10} - \frac{15}{51}(M'_{10} - M'_{12}) + \frac{15}{51} \times \frac{11}{55}(M'_{21} - M'_{23})$$

$$M_{12} = M'_{12} + \frac{36}{51}(M'_{10} - M'_{12}) + \frac{15}{51} \times \frac{11}{55}(M'_{21} - M'_{23})$$

$$M_{21} = M'_{21} - \frac{33}{51} \times \frac{20}{55}(M'_{10} - M'_{12}) - \frac{18}{51}(M'_{21} - M'_{23})$$

$$M_{23} = M'_{23} - \frac{33}{51} \times \frac{20}{55}(M'_{10} - M'_{12}) + \frac{33}{51}(M'_{21} - M'_{23}),$$

which is identical with the equations derived from the author's analysis.

It will be noticed that in the above equations there is no indication of any influence on any of the end moments from a rotation of joints 0 and 3 (A and D in the author's designation). These extreme ends of the system may be either fixed or hinged. In either case the rotation at the ends resulting from a load on the system need not be considered. It is recognized, however, that an analysis of unbalanced moments at these points under a condition of hinged ends could serve a useful purpose in case of a cantilever overhang beyond either of these ends. The author probably had such a case in mind in thus extending his analysis, although he did not specifically state so.

Incidentally, since the author leans entirely on his illustration to develop his method of analysis, it is regrettable that he limited his main illustration to a single case with hinged ends. An additional similar analysis of a system with fixed ends would have been appreciated by the reader. However, the writer attempted to follow through a similar analysis of the same example under a condition of fixed ends and obtained the following end moments:

$$M_{10} = M'_{10} - \frac{6}{17}(M'_{10} - M'_{12}) + \frac{1}{17}(M'_{21} - M'_{23})$$

$$M_{12} = M'_{12} + \frac{11}{17}(M'_{10} - M'_{12}) + \frac{1}{17}(M'_{21} - M'_{23})$$

$$M_{21} = M'_{21} - \frac{4}{17}(M'_{10} - M'_{12}) - \frac{5}{17}(M'_{21} - M'_{23})$$

$$M_{23} = M'_{23} - \frac{4}{17}(M'_{10} - M'_{12}) + \frac{12}{17}(M'_{21} - M'_{23})$$

which are identical with those obtained from the writer's own formulas under similar conditions.

DAVID V. MESSMAN,[10] M. ASCE—The author's analysis of the effect of multiple loadings on a frame is convenient but has the disadvantage of combining moments from joint rotation and those from carryovers in such a way as to permit errors not easily detectable without careful checking.

Another method is available, similar in permitting a single distribution but showing the moments from rotation separate from those from carryovers. To describe this method an analysis of the same three-span problem used by the author is illustrated by the writer's Figures 1, 2, and 3.

In the writer's Figure 1 apply an arbitrary external moment at B to produce distributed moments shown, and then an external moment at C to produce carryover moment at B such that no external moment is required to hold joint B. The external (or unbalanced) moment at C is indicated in terms of a fixed end moment at C, of -51 on D side of C, although it could be shown as +51 on B side of joint C just as well. This influence table is for unbalance at C, but an influence table for unbalance at B would also serve.

In the writer's Figure 2, the starting fixed end moments are the same as those used by the author. All unbalance is collected at C, the same point having unbalance in the writer's Figure 1, by rotating A to free joint A and then rotating joint C to balance joint B.

The writer's Figure 3 is the summarized solution, with quantities being computed in the order indicated by lower case letters. The starting point is the quantity +35.3 which is derived by comparing the unbalance in the writer's Figures 1 and 2. The writer's Figure 3 represents a complete and practically self checking solution, since errors in previous work will prevent joints from balancing. Furthermore, a check of the work need concern this figure only, since any set of figures which balances the joints and represents correct ratios of moments from distribution and carryovers is the one and only solution.

For other loadings the writer's Figure 1 would be used without change.

The author speaks of the complications of direct analysis of frames having a closed path. As a matter of possible interest, an application of the writer's method is shown below for the analysis of a frame, writer's Figure 4, having a single closed path, one loading of which produces the fixed end moments shown, sign designations being positive for tension on inside face of members, with quantities in parentheses being relative stiffnesses.

Influence tables for unbalanced moment at joint A of the frame are computed as illustrated by the writer's Figures 5, 6 and 7, in which lower case letters indicate order of computation. By Maxwell's Theorem of Reciprocal Displacements, the rotation of the B side of joint A in the writer's Figure 5 is of the same magnitude and direction as the rotation of the D side of joint A in the writer's Figure 6, since the starting carryover moments of 6 chosen for the D side and B side in the two figures are numerically equal and act on the joint in the same rotational direction. The writer's Figure 7 is obtained by combining the writer's Figures 5 and 6, allowing joint A to be reconnected, and collecting all unbalanced moment (-1866) on one side for convenience. Quantities at other joints are not needed.

In the writer's Figure 8, joints of the frame are rotated to collect all unbalanced moment of the given loading at the one joint for which the influence table has been made, in this case joint A, cutting joint A in the process.

The writer's Figure 9 shows a complete solution for the B side of Joint A, values therefor being combined with those in the writer's Figure 8 for

10. Eng. of Bridges, Central Lines, Southern Railway System, Knoxville, Tenn.

use as a starting joint for the summarized solution of all joints in the writer's Figure 10.

As is the case in the writer's Figure 3 for the solution of the 3-span continuous beam, the writer's Figure 10 is practically self-checking, and is a self-contained complete solution.

For other loadings applied to this frame the writer's Figures 5, 6 and 7 could be utilized without change.

	A	B		C		D
Relative Stiffnesses	.75*		2		3*	
Fixed End Moments	0	0	0	0	-51	0
Distributed Moments	0	+3	-8	-22	+33	0
Carryovers	0	0	+11	+4	0	0
Final Moments	0	+3	+3	-18	-18	0

* adjusted for free end

Messman on Looney, Figure 1 - Influence Table

	A (.75)	B	(2)	C	(3)	D
F.E.M. (A fixed)	-100	-100	0	0	0	0
D.M.	+100	0	0	0	0	0
C.O.	0	-50	0	0	0	0
F.E.M. (A free)	0	-150	0	0	0	0
D.M.	0	0	0	+300	-450	0
C.O.	0	0	-150	0	0	0
F.E.M. (U.B.M.@C)	0	-150	-150	+300	-450	0

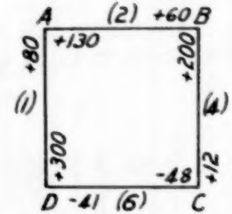
-750 = U.B.M.

Messman on Looney, Figure 2

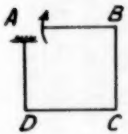
	A (.75)	B	(2)	C	(3)	D
F.E.M. (A fixed)	-100	-100	0	0	0	0
D.M.	+100	0	0	0	0	0
C.O.	0	-50	0	0	0	0
F.E.M. (A free)	0	-150	0	0	0	0
D.M.	0	(i) +44.1	-117.6 (f)	(c) -23.5	+35.3 (a)**	0
C.O.	0	0	+11.8 (g)	(e) +58.8	0	0
Final Moments	0	(j) -105.9	-105.8 (h)	(d) +35.3	+35.3 (b)	0

$$** +35.3 = \frac{-750\ (+33)}{-51} - 450$$

Messman on Looney, Figure 3 - Summary



Messman on Looney, Figure 4



	A	(2)	B	(4)	C	(6)	D	(1)	A
F.E.M.	0	-1224 (x)	0	0	0	0	0	0	0
D.M.	0	+1352 (u)	(r) +244	-488 (a)	(l) -112	+168 (i)	(f) +72	-12 (c)	0
C.O.	(a) +6	-122 (v)	(t) -676	+56 (p)	(n) +244	-36 (j)	(h) -84	0 (d)	(a) +6
F.M.	(b) +6	+6 (w)	(s) -432	-432 (q)	(m) +132	+132 (k)	(g) -12	-12 (e)	(b) +6

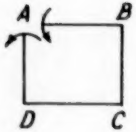
Messman on Looney, Figure 5- Influence Table, cut at A, fixed toward D.

A	(2)	B	(4)	C	(6)	D	(1)	A
(x) +642	0	0	0	0	0	0	0	(x) +642
(u) -676	0	(c) +12	-24 (f)	(i) -72	+108 (l)	(o) +336	-56 (r)	(u) -676
(v) +28	-6 (a)	(d) 0	+36 (h)	(j) +12	-168 (n)	(p) -54	+338 (t)	(v) +28
(b) -6	-6 (b)	(e) +12	+12 (g)	(k) -60	-60 (m)	(q) +282	+282 (s)	(w) -6

Messman on Looney, Figure 6- Influence Table, cut at A, fixed toward B.

D (1)	A	(2) B
F.E.M.	0	-1866
D.M.	-676	+1352
C.O.	+34	-128
F.M.	-642	-642

Messman on Looney, Figure 7- Influence Table for unbalance at A.



	A (2)	B	B (4)	C	C (6)	D	D (1)	A
F.E.M., from Fig. 4	+130	+60	+200	+12	-48	-41	+300	+80
D. M.	-280(d)	0	0	0	0	(i) -120	+ 20 (k)	(n) +962
C. O.	0	(c) +140	0	0	+60 (h)	0	-481 (m)	(o) -10
F.M. (U.B.M. at A)	-150(e)	(b) +200	+200(a)	(f) +12	+12 (g)	(j) -161	-161 (l)	(p) +1032

Messman on Looney, Figure 8 - Joint A cut, all joints balanced except A.

	D (1) A	A (2) B	
F.E.M.	+1032	-150	, from Fig. 8, with joint A cut.
D.M.	0	-1644	$= (-2 \times 962) - (-280)$, required to reconnect A.
C.O.	$\frac{(-1644)(+6)}{1352} = -7.30$	$+148.35 = \frac{(-1644)(-122)}{1352}$	, see Fig. 5.
F.E.M.	+1024.70	-1645.65	, with joint A reconnected, U.B.M. = -2670.35 on B side.
D.M.		$+1934.79 = \frac{(-2670.35)(1352)}{-1866}$	, see Figure 7.
C.O.		$-183.18 = \frac{(-2670.35)(-128)}{-1866}$	, see Figure 7.
Final Moment		+105.96	

Messman on Looney, Figure 9 – Solution for B side of joint A.

		A (2)	B	B (4)	C	C (6)	D	D (1)	A
	F. E. M., from Fig. 4	+130	+60	+200	+12	-48	-41	+300	+80
D. M.	-280 -1644 +1934.79 →	+ 10.79 (a)	(d) +69.66	-139.32 (g)	(j) -127.16	+190.74 (m)	(u) +376.32	- 62.72 (r)	(o) - 5.40
C. O.	-183.18 + 148.35 →	- 34.83 (b)	(e) -5.40	+63.58 (i)	(k) +69.66	-188.16 (w)	(n) -95.37	+ 2.70 (s)	(q) +31.36
	Final Moments	+105.96 (c)	(f) +124.26	+124.26 (h)	(l) -45.50	-45.42 (x)	(v) 239.95	+239.98 (t)	(p) +105.96

Messman on Looney, Figure 10 - Summary.

Discussion of
CALCULATION OF PRESSURE OF CONCRETE FORMS

by R. Schjödt
(Proc. Paper 680)

J. J. POLIVKA,[1] M. ASCE—The safety and the economy of formwork is important for concrete structures poured in place. Besides the formwork should ensure the designed shapes and dimensions and its deflections and deformations should be avoided as much as possible. For this reason all factors affecting these requirements, e.g. strength and elasticity of materials used, structural design of the formwork, methods of pouring and of compaction or vibration of concrete and its physical and chemical properties including methods of curing should be considered. Any contribution to improve the basic properties of the formwork are therefore very valuable and important for broader use of this excellent structural material. As in any technical problems the work which has been previously done should be known for better solution and improvements and the writer wishes to add some further references:

12. Raymond E. Davis and Harmer E. Davis, Compaction of Concrete Through the Use of Vibratory Tampers, J. ACI, June '33.
13. A. E. Wynn, Design and Construction of Formwork for Concrete Structures, Concrete Publ., Ltd., London.
14. A. B. MacMillan, How to Save in Concrete Work, Concrete, Jan. to Dec., '30.
15. Portland Cement Association, Pressure of Concrete on Formwork, No. ST3, '42.
16. H. Muhs, Measurements of Pressure on Forms for Heavy Concrete structures, Beton-u. Stahlbeton, No. 7, '51.
17. The Design of Forms and Falsework for Concrete Bridge Construction, Manna Roads Journal, Sept., '47, Sydney.
18. Toussaint, Pressure on Forms for Large Concrete Members, Oesterr. Bauz., No. 1, '50.
19. H. Muhs, Pressure on Concrete Forms. Beton-u. Stahlbetonbau, June, '55.

The latter paper is of special interest since the author expresses theoretically the results of a series of tests, which is the same aim of the paper by R. Schjödt.

CORRECTIONS—On page 680-5, in Eq. 11 the integrand should be changed from $e = h^2$ to e^{-h^2}. On page 680-6, the term in the third line from the bottom that reads $\frac{1}{3\phi}$ should be changed to $\frac{1}{3}\phi$. On page 680-7, the computation for p should have a term 62.1 added before the second equal sign. On page 680-8, the computation for p should have a term 62.5 added before the second equal sign.

1. Cons. Eng., Berkeley, Calif.

Discussion of
BENDING IN ANNULAR SECTIONS

by Adolphe A. Marrone
(Proc. Paper 683)

ABRAHAM I. GARTNER,[1] A.M. ASCE—The analysis of annular sections by aid of the properties of circular arcs on the assumption that the annulus is very thin has been treated in a number of sources[2,3]. Nevertheless, the author of this paper is to be highly commended for his extensive labor in constructing a number of charts, which will be of great help to the designer in applying this method. It is perhaps to be regretted that he has chosen to base his curves on the argument α rather than the diametral ratio k, which is more generally used in design.

Of particular value are the correction curves for the true stress at the extreme edge of the annulus. Usually this correction is made solely on the basis of the ratio of distances from the neutral axis. Whereas the author's curves take also into account the difference resulting from the assumption that the area under compression is a sector, while in reality it is a segment of the annular ring.

From the shape of these curves it is readily apparent that as soon as t/R exceeds 0.2 or α comes to less than 60° (k 0.25), the necessary correction increases very rapidly.

Still, the procedure given in this paper, no different from other sources, is essentially one of investigation, requiring several trials before a satisfactory design is achieved. The writer, being much interested in the design of anchorages for tall cylindrical structures under heavy lateral loading, has constructed two nomograms to facilitate this operation. The first is used to find directly the amount of anchor bolt steel (or reinforcement) required for any assumed concrete section and from the given allowable stress in steel; the second is intended for investigating the stresses in existing designs.

It is believed that these nomograms, reproduced hereby, are capable of much wider application than the purpose for which they were originally published.[4] Directions for their use, and the meaning of the symbols employed are given on the charts. The following few remarks will serve to explain them further.

1) In problems of this nature the thickness of the concrete ring is generally determined from other considerations, and it is seldom that maximum concrete stresses are utilized. In Nomogram I., as long as k falls below the line for the given working stress in steel, the concrete stress will be well within the allowable. For concrete up to 3000 psi ultimate, these lines represent the point where the average bending stress in the annulus is 33% of

1. Contract Supervisor, Foster-Wheeler Corp., New York, N.Y.
2. F. W. Taylor, S. E.Thompson and E. Smulski, Concrete-Plain and Reinforced, John Wiley and Sons, New York.
3. George A. Hool and W. S. Kinne, Reinforced Concrete and Masonry Structures, McGraw-Hill Co., New York.
4. "Nomograms for the Solution of Anchor Bolt Problems", Petroleum Refiner, vol. 30, pp. 101-106. July 1951.

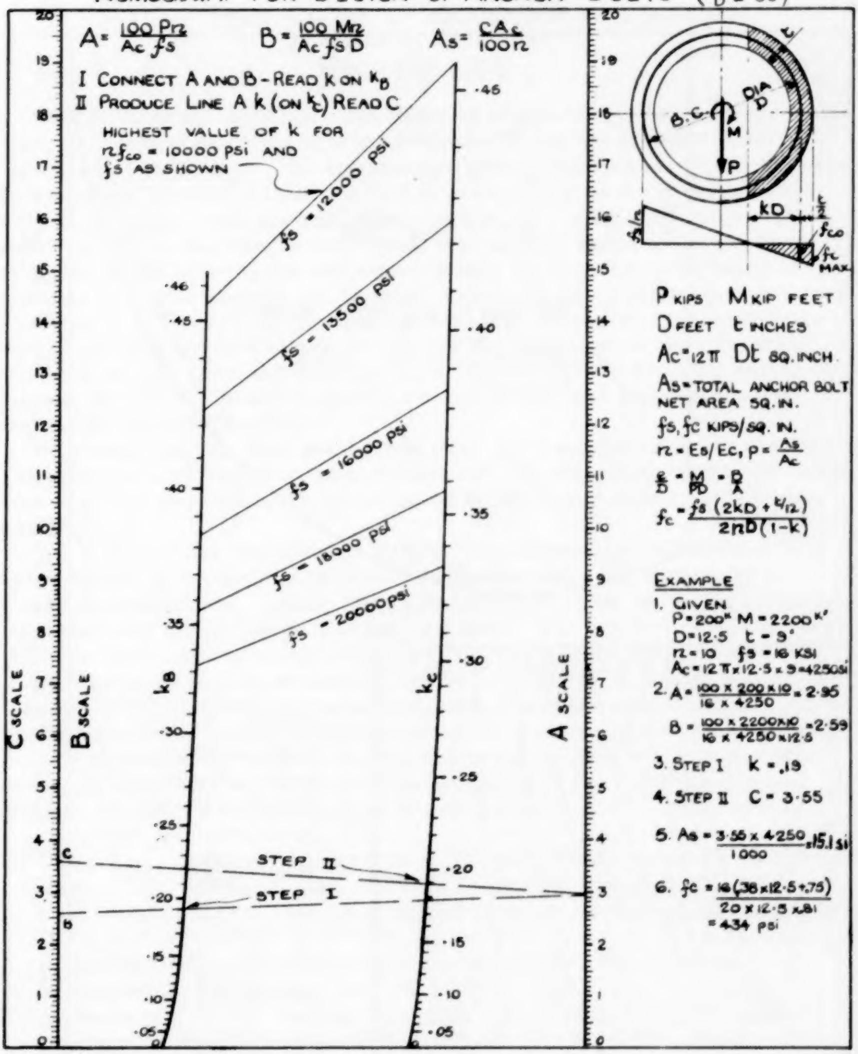
the ultimate, leaving sufficient margin for the necessary correction to the extreme edge. If k falls above the appropriate line, the concrete stress should be computed and, if necessary, the section increased. A similar series of lines may be drawn for concrete of higher ultimate strength.

2) The nomogram can be used for pure bending without direct stress by setting $A = 0$.

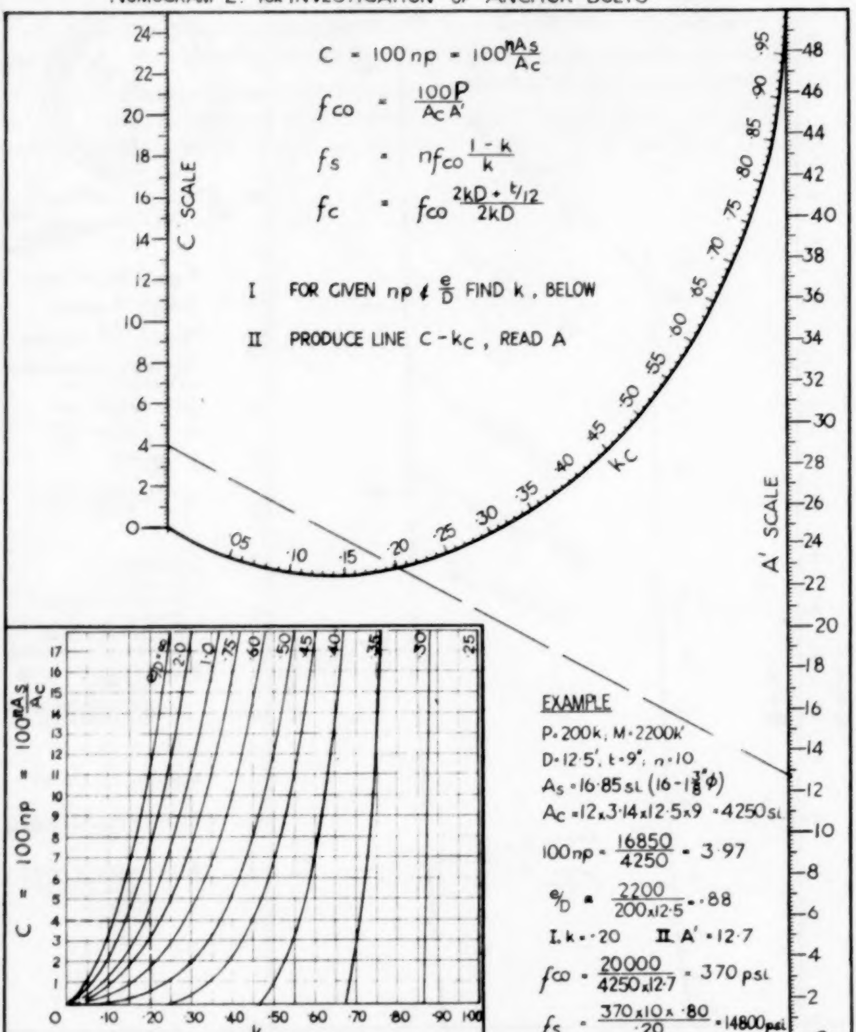
3) If it is desired to consider the value of the steel in compression, the quantity A_c should be increased by the estimated value of nA_s.

4) The C scale (as well as the A and B scales) is linear; consequently, if in step 2 the straight line intersects the C scale outside the limits of the chart, the value on the latter can be easily extrapolated.

NOMOGRAM I. FOR DESIGN OF ANCHOR BOLTS ($\frac{e}{D} \geq .50$)



NOMOGRAM 2. FOR INVESTIGATION OF ANCHOR BOLTS



Discussion of

ECONOMY AND SAFETY OF DIFFERENT TYPES OF CONCRETE DAMS

by August E. Komendant
(Proc. Paper 684)

J. J. POLIVKA,[1] M. ASCE—The paper is of special interest not only to designers of larger dams but also to authorities in charge of water power, irrigation and water supply for agricultural and domestic need. These projects are in public interest so that practically every citizen and tax payer is concerned with their economy and safety. The paper should remind the engineers and the authorities to investigate in each individual case those important factors. Additional fee and expenses may be involved by the study of possible alternate designs, but in most cases millions of dollars can be saved to the public. The author mentions also the high economy of constant-angle dams, invented by Lars Jorgensen, M. ASCE. After Jorgensen's death the writer took over his activity in this field. He was able to estimate saving of approx. 36 million dollars in construction of 61 dams of Jorgensen's type built in the period 1912-1950.[2]

It is clear that the "leaf spring type shell dam" originated by the author and professor Dischinger is very economical. As the author states, the basic idea of such a dam was already conceived by Maillart,[3] Mesnager,[4] Noetzli and others.[5]

The author states that straight gravity dams regardless of tremendously large volume of concrete have relatively small safety, due especially to stress concentrations. Linear stress distributions under the water pressure combined with the proper weight does not exist. The stress analysis is very intricate, both theoretical and experimental. It should be remembered that H. M. Westergaard, consulting engineer for the Hoover Dam, was able to determine by his slab-analogy method only the stresses inside the dam.[6] However, it was found later by photoelastic stress analysis by the writer that in certain horizontal sections of the dam the boundary stresses increase dangerously, especially due to stress concentrations at the reentrant corners between the body of the dam and the foundation block.[7] The tests refer to

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1. Cons. Eng., Lecturer, Stanford Univ., Stanford Calif.; formerly Research Assoc., Eng. Materials Lab., Univ. of California, Berkeley, Calif.
2. J. J. Polivka: Large savings estimated from constant-angle dam design, CIVIL ENGINEERING, Nov. 1950.
3. R. Maillart, Gewoelbe-Staumauern mit abgestuften Druckhoehen, Schweizerisch Bauzeitung, Apr. 14, 1928, p. 183.
4. A. Mesnager, Sur les barrage-reservoirs à voûtes et à charge fractionnée, Revue générale de l'électricité, Oct. 29, 1927, p. 681.
5. A. Floris, Dams with stopped Water levels, Western Constr. News, Oct. 25, 1928, p. 666.
6. Bureau of Reclamation, Slab analogy experiments. Boulder Canyon Projects Final Reports, Part V, Bulletin 2, Denver, Colo., 1938.
7. J. J. Polivka, Analysis of Gravity Load Stresses by Photoelastic Methods, Proc. of the 16. Semi-annual Photoelasticity Conference, Ill. Inst. of Techn., '42.

gravity forces and the experimental stresses were incorporated ("frozen") into a bakelite model in a centrifuge oven by slow annealing during rotation of about 1000 RPM, and prove that the gravity stresses in critical sections considerably exceed the average values. Usually, the gravity dams are built in arched shape which increases structural resistance, otherwise some gravity dams would be still less economical.

It may be also of interest to investigate and to compare the economy and the safety of steel dams built in the U.S. and elsewhere at the beginning of this century[8] which were 30 to 50 per cent cheaper than concrete dams (e.g. competitive comparison with concrete dam across the Saranac River at the Kent's Fall, Slaguay dam, Col., Ash Fork dam, Arizona, Redridge dam, Mich.). The construction of steel dams was abandoned because of corrosion which disadvantage, at present, can be successfully prevented by protective coatings and new metal alloys. Also new, more economical structural schemes were proposed recently, as e.g. shell type dam invented and patented by prof. G. Krivosheine, J. Fultner and Dr. Sekla,[9] represented in the U.S. by the writer.[10]

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8. Hovey O.L., Steel Dams, American Inst. of Steel Constr., N. Y., '35; Wegmann, Design and Construction of Dams, John Wiley & Sons; Bainbridge F.H., Structural Steel Dams, J. West Soc. Engrs., vol. 8, 1903; articles in Eng. News Rec.: Four Steel Dams-Their Design and History, by J. F. Jackson, '30, Why Not Steel Dams? by C. M. Stanley, '32, Force of Habit, Editorial, '32.
9. Krivocheine, G., Steel Dams, 2nd Congress of Internat. Assn. for Bridge and Struct. Eng., '38.
10. The writer collaborated with J. Fultner on design of a steel dam on the Svratka River, Brno, Moravio, for which the Vitkovice Steel Works were bidding, in 1934, 5,275,612.-Cz. Cr. as compared with the average bid of 15 contractors for the concrete alternate: 9,942,529.- Cz. Cr.

Discussion of
TIGHTENING HIGH STRENGTH BOLTS

by F. P. Drew
(Proc. Paper 786)

ADRIAN PAUW,* A.M. ASCE—The purpose of all methods of tension control for high strength bolts is to develop the necessary minimum clamping force which is required with the use of this type of fastener. In order to insure that this minimum clamping force would be developed, the Research Council on Riveted and Bolted Joints in its specification for "The Assembly of Structural Joints Using High Strength Steel Bolts" recommended a "target" value of 15% in excess of the required minimum bolt tension (90% of the specified elastic proof load). The "turn of the nut" procedure described by Mr. Drew is a practical and simple procedure for achieving the minimum clamping force required; its use, however, entails the abandonment of the concept of a "target" value for the clamping force, since bolts tightened by this procedure will generally be stressed well beyond the elastic limit. Field experience and laboratory tests indicate that stressing bolts beyond the elastic limit during assembly is not necessarily undesirable, indeed, where fatigue stresses are involved, tests indicate that the development of the maximum clamping capacity is beneficial.

There are, however, two questions which this discussor believes warrant further study:

1. Can the adoption of the "turn of the nut" procedure, under certain conditions, lead to structural damage of the bolt, resulting in failure of the bolt or loss of clamping force?

2. If the concept of an "optimum target value" for the clamping force is to be abandoned, does the "turn of the nut" procedure necessarily provide the safest and simplest method for tension control?

During the past year Mr. L. L. Howard conducted a statistical study at the University of Missouri embodying test of about 1200 high strength structural bolts tightened with pneumatic impact wrenches. Toward the end of his testing program, at the suggestion of Mr. Drew, the number of turns of the nut were recorded for 166 bolts in the last series of tests. In all of Mr. Howard's test, bolts were tensioned against the abutments of a calibrated dynamometer, instrumented to permit recording of the clamping force during the tensioning operation. Some interesting information was revealed which generally confirms Mr. Drew's observations:

1. Tension values for the 7/8 inch bolts tested were obtained with 0.6 to 0.9 revolutions of the nut. All tension values observed were at least equal to the minimum specified elastic proof load. (35.97 kips)

2. The tightening studies on the 3/4 inch bolts covered a wider range of nut revolutions—from 0.8 to 1.5. All tension values were substantially higher than the minimum specified elastic proof load and a number of them (all

* Associate Prof., Dept. of Civ. Eng., Univ. of Missouri, Columbia, Missouri.

associated with more than one revolution of the nut) exceeded the minimum specified ultimate proof load.

3. When bolts are loosened and retensioned, the same tensions are achieved on the second trial as on the first with approximately one quarter less turn of the nut.

4. After the nut is tensioned to produce nearly the full ultimate proof load, this tension value is maintained through quite a large amount of nut revolution.

Following Mr. Howard's original tests, several additional tests were made using an automatic "turn of the nut" recording device. A typical set of tension-time, nut revolution-time curves is shown in Fig. 1. For this test the air pressure of the impact wrench was adjusted to give the maximum clamping force in about ten seconds of impacting. The recommended minimum was achieved in about three seconds, while about 20 seconds were required to turn the nut one complete revolution. It should also be noted that the maximum clamping force was maintained over a large impacting time interval; when the nut had been turned through about 1.3 revolutions the clamping force gradually decreased. The upper half of Fig. 2 shows an expanded portion of the critical region of the curves shown in Fig. 1. It should be noted that although the tension-time, nut revolution-time curves are similar in shape, the ordinates of the latter continue to increase at a fairly constant rate after the maximum clamping force has been reached.

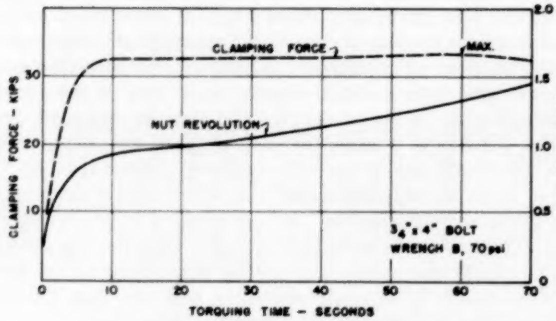
The lower portion of Fig. 2 shows the clamping-force, nut-revolution relationship for several bolts tightened at three different air pressures. Note that after the initial "take up" the clamping-force, nut revolution relationship is essentially linear up to the elastic limit of the bolts. It is the variability in the initial "take-up" which presents one of the principal drawbacks in the "turn of the nut" method. For this method the starting point is specified as "finger-tight". Bolt threads are apt to have slight imperfections requiring the use of a wrench in "making up" the bolt assembly. Even with a small wrench a 1/4 turn is readily obtained. An additional full turn of the nut may then produce such strains in the bolt that further tightening of the nut results in a reduction rather than an increase in the clamping force. Another factor which affects the initial starting point is the slope of the abutment surface under the nut. Slopes of one in twenty are permissible without the use of bevelled washers, thus about a half turn is required to bring the nut into full bearing. Under this condition one full turn of the nut may not be sufficient to develop the minimum specified clamping force.

Another factor which may affect the reliability of the turn of the nut method and which deserves further investigation is the effect of the grip length and the number of exposed threads under the nut. The greater the grip and the more plies, the greater the pick up in the steel. Also, since bolts are strained into the plastic region, the number of exposed threads under the nut is of importance. Conceivably, a short grip bolt with very little pick up in the steel and with only a few exposed threads could be stretched sufficiently to throw the threads of lead.

One of the economies to be derived from the use of high strength bolts is that separate fitting up bolts can be eliminated. The turn of the nut method produces a permanent set equivalent to about a half turn. Retightening of these bolts with a full turn of the nut may then produce excessive strains, causing failure by stripping or rupture, or a reduction in the clamping force. The effect of retightening bolts is shown in Fig. 3. In this Figure, bolt

tension versus torquing time curves are shown for five pressure increments. This Figure brings out several interesting factors. First, it should be noted that the operating pressure of the impact wrench is not particularly critical; the specified minimum clamping force was developed in a reasonable time for all but the lowest pressure increment. The second factor of interest is that the clamping force is developed more rapidly when bolts are retightened. This phenomenon may be explained due to a "running in" effect of the threads during the first tightening. This also accounts for the fact that minimum bolt tensions were developed with about a quarter less turn of the nut during retensioning. It should also be noted that a point may be reached where further torquing of the nut results in a more or less gradual reduction in the clamping force. When bolts fail due to thread stripping rather than by suddenly twisting off, there is no visual indication that some of the clamping force has been lost due to excessive tightening. If bolt tension is achieved by the method of controlling air pressure and torquing time, then it is quite apparent that with a "target" clamping force approaching the maximum value developable by torquing, a large range of both air pressure and torquing time may be employed without incurring the danger of excessive tightening.

The problem of bolt tensioning is essentially a problem of quality control. Applied with a little common sense satisfactory results should be obtainable either with the turn of the nut procedure or with the method of controlling pressure and time of impacting. A combination of the two methods may be possible, whereby the need for special devices for field calibration of impact wrenches is eliminated and only an occasional check of the nut revolution during tightening is used to check the accuracy of the wrench calibration.



TENSION-TIME & NUT REVOLUTION-TIME CURVES

Fig. 1

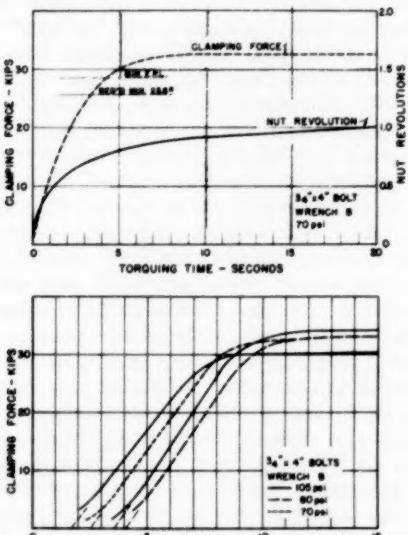
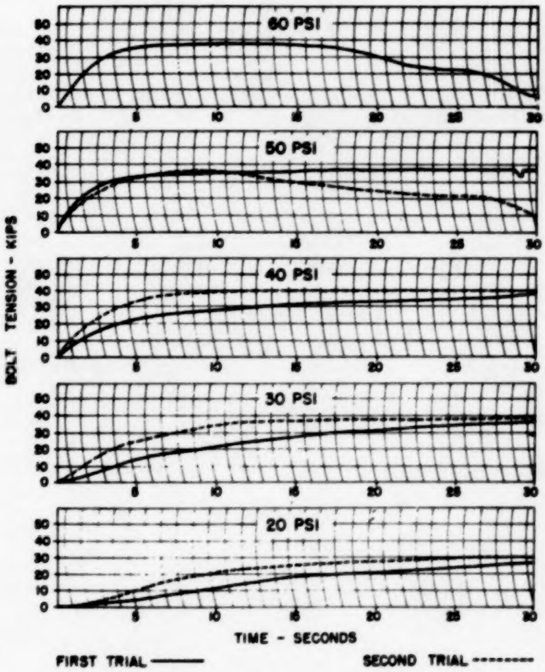


Fig. 2



TIME VERSUS BOLT TENSION
TYPE AR $\frac{3}{4}$" ϕ x 4"

Fig. 3

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